

arXiv:0707.4554 [hep-th]

August 7, 2007

OU-HET 583

Exact Solutions and the Attractor Mechanism in Non-BPS Black Holes

Kyosuke Hotta ¹ and Takahiro Kubota ²

*Graduate School of Science, Osaka University,
Toyonaka, Osaka 560-0043, Japan*

Abstract

The attractor mechanism for the four-dimensional $\mathcal{N} = 2$ supergravity black hole solution is analyzed in the case of the D0-D4 system. Our analyses are based on newly derived exact solutions, which exhibit explicitly the attractor mechanism for extremal non-BPS black holes. Our solutions account for the moduli as general complex fields, while in almost all non-BPS solutions obtained previously, the moduli fields are restricted to be purely imaginary. It is also pointed out that our moduli solutions contain an extra parameter that is not contained in solutions obtained by replacing the charges in the double extremal moduli solutions by the corresponding harmonic functions.

¹hotta@het.phys.sci.osaka-u.ac.jp

²kubota@het.phys.sci.osaka-u.ac.jp

1 Introduction

It has been pointed out that supersymmetric (SUSY) black hole solutions exhibit a peculiar property called the attractor mechanism [1]-[6]. It has been confirmed in the case of extremal black holes that moduli fields are drawn to some fixed values at the horizon of the black holes, independently of their asymptotic values. In other words, the fixed values of the moduli at the horizon are characterized only by the charges carried by the black holes. This fact has been studied by using the BPS attractor equations, and an algorithm for calculating the macroscopic Bekenstein-Hawking entropy has been established. As a result, it has been found that the entropy is given by the extremum value of the central charge [2].

In the last several years, the study of the attractor mechanism has been extended to non-supersymmetric cases [7]-[18], [21]. Many of the properties of attractive BPS configurations seem to be shared by non-BPS attractor configurations, provided that the solutions are extremal. A non-BPS attractor equation has been constructed to relate the charges to the attractive values of the moduli [12]. Although the attractor equation is very useful, the most direct way to examine the nature of black hole solutions is to obtain the solution for the moduli fields in the whole space. However, the analytic approach to obtaining solutions in non-BPS cases is complicated, because in such cases, it is necessary to deal with second-order differential equations, while the BPS equations in SUSY cases are first order. In [7], a perturbative method is applied to extremal black holes, and it is found that the attractor is effective. Numerical results support these perturbative results.

It is pointed out in [5], [6] and [19] that in the BPS case, the exact supersymmetric solutions of moduli fields in $\mathcal{N} = 2$ supergravity coupled to an arbitrary number of vector multiplets may be obtained from the double extremal solutions by simply replacing the charges by harmonic functions. In the case of non-BPS black holes with D2 and D6 brane charges, Kallosh et al. [14] remarked that the exact solution in the STU model can be obtained using the same procedure of replacing the charges in the double extremal moduli solutions by harmonic functions. Their solutions are, however, still restricted, because the moduli fields are not general complex numbers but, rather, purely imaginary ones.

The purpose of the present paper is to derive general exact solutions in four-dimensional $\mathcal{N} = 2$ supergravity (from the Type IIA superstring) coupled to vector multiplets for the case of a non-BPS extremal black hole with D0-D4 brane charges. Our exact solutions obtained for the STU model are more general than those obtained by previous authors for the D0-D4 system, because the moduli fields are not restricted to purely imaginary values, but, instead,

are general complex numbers. As it turns out, the initial conditions of our solutions have more degrees of freedom than those obtained by simply replacing the charges in the double extremal moduli solutions by the corresponding harmonic functions. The additional degrees of freedom come from the arbitrary complex values that we choose for the asymptotic values of the real parts of the moduli fields.

In a recent paper [18], Saraikin and Vafa obtained non-BPS double extremal black hole solutions, treating the scalar fields as general complex numbers. It is a very intriguing question whether the prescription of replacing the charges at the horizon in their solution by the corresponding harmonic functions leads to general solutions in the whole space. For the BPS case, it has been shown that this prescription is in fact effective for obtaining general solutions (see Appendix A for more details). It is beyond the scope of the present paper to give an answer to the above question, but the explicit non-BPS extremal solutions derived in this paper could be useful for investigating such questions.

2 $\mathcal{N} = 2$ supergravity

We study $\mathcal{N} = 2$ supergravity coupled to $(N_V + 1)$ vector multiplets. The bosonic part of the Lagrangian is given by

$$8\pi e^{-1}\mathcal{L} = -\frac{1}{2}R - G_{a\bar{b}}\partial_\mu z^a \partial^\mu \bar{z}^b + \frac{i}{4}(\bar{\mathcal{N}}_{IJ}F_{\mu\nu}^{I-}F^{J-\mu\nu} - \mathcal{N}_{IJ}F_{\mu\nu}^{I+}F^{J+\mu\nu}). \quad (2.1)$$

Here, we define the moduli fields

$$z^a = \frac{X^a}{X^0}, \quad (a = 1, 2, \dots, N_V) \quad z^0 = 1 \quad (2.2)$$

in terms of the complex scalar field X^I (where $I = 0, 1, \dots, N_V$) of vector multiplets. For the sake of simplicity, we set Newton's constant to unity. As usual, the Kähler metric, $G_{a\bar{b}}$, is defined in terms of the Kähler potential $K(z, \bar{z})$,

$$e^{-K(z, \bar{z})} = -z^I N_{IJ} \bar{z}^J = |X^0|^{-2}, \quad (2.3)$$

where N_{IJ} is related to the second derivative of the prepotential F as

$$N_{IJ} = 2 \operatorname{Im} F_{IJ} = 2 \operatorname{Im} \frac{\partial^2 F(X)}{\partial X^I \partial X^J}. \quad (2.4)$$

More explicitly, $G_{a\bar{b}}$ is written

$$G_{a\bar{b}} = \partial_a \partial_{\bar{b}} K = -\left(z^K N_{KL} \bar{z}^L\right)^{-1} N_{ab} + \left(z^K N_{KL} \bar{z}^L\right)^{-2} N_{aI} \bar{z}^I N_{bJ} z^J. \quad (2.5)$$

In (2.1), we have also introduced the quantity

$$\mathcal{N}_{IJ} = \overline{F}_{IJ} + i \frac{N_{IK} z^K N_{JL} z^L}{z^M N_{MN} z^N}, \quad (2.6)$$

which is also a function of the moduli. The real and imaginary parts of \mathcal{N}_{IJ} are denoted by ν_{IJ} and μ_{IJ} , respectively; i.e., we have

$$\mathcal{N}_{IJ} = \nu_{IJ} + i\mu_{IJ}. \quad (2.7)$$

Let us begin with the static metric

$$ds^2 = -e^{2U(\tau)}(dt)^2 + e^{-2U(\tau)} \left\{ \frac{1}{\tau^4} (d\tau)^2 + \frac{1}{\tau^2} d\Omega^2 \right\}, \quad (2.8)$$

where the horizon corresponds to $\tau \rightarrow -\infty$ and the spatial infinity to $\tau \rightarrow 0$. [This metric corresponds to the case of extremal black holes, where the non-extremality parameter c has been set equal to zero in (B.2).] To solve the equations of motion for $U(\tau)$ and z^a coupled to the gauge fields, we postulate

$$F_{tr}^I = \tilde{q}^I, \quad F_{\theta\phi}^I = p^I \sin\theta, \quad G_{Itr} = \tilde{p}_I, \quad G_{I\theta\phi} = q_I \sin\theta, \quad (2.9)$$

where the magnetic fields are defined as $G_{I\mu\nu}^- = \overline{\mathcal{N}}_{IJ} F_{\mu\nu}^{J-}$ and \tilde{q}^I and \tilde{p}_I are given by the electric and magnetic charges q_I and p^I of the black hole as

$$\tilde{q}^I = e^{2U} [(\mu^{-1})^{IJ} \nu_{JK} p^K - (\mu^{-1})^{IJ} q_J], \quad (2.10)$$

$$\tilde{p}_I = e^{2U} [\nu_{IJ} (\mu^{-1})^{JK} \nu_{KL} p^L - \nu_{IJ} (\mu^{-1})^{JK} q_K + \mu_{IJ} p^J]. \quad (2.11)$$

With the gauge field configurations appearing in (2.9), the equations of motion turn out to be

$$U'' = e^{2U} V_{BH}, \quad (2.12)$$

$$- \{U'' - 2(U')^2\} + 2G_{a\bar{b}}(z^a)'(\bar{z}^b)' - e^{2U} V_{BH} = 0, \quad (2.13)$$

$$\{G_{a\bar{b}}(\bar{z}^b)'\}' - \partial_a G_{b\bar{c}}(z^b)'(\bar{z}^c)' = e^{2U} \partial_a V_{BH}. \quad (2.14)$$

Here, the black hole potential V_{BH} is given by

$$V_{BH}(z, \bar{z}, p, q) = -\frac{1}{2}(p^I, q_J) \begin{pmatrix} (\nu\mu^{-1}\nu + \mu)_{IK} & -(\nu\mu^{-1})_I{}^L \\ -(\mu^{-1}\nu)^J{}_K & (\mu^{-1})^{JL} \end{pmatrix} \begin{pmatrix} p^K \\ q_L \end{pmatrix}. \quad (2.15)$$

Note that the equations of motion (2.12)–(2.14) can be derived from the Lagrangian

$$\mathcal{L}(U(\tau), z(\tau), \bar{z}(\tau)) = (U')^2 + G_{a\bar{b}}(z^a)'(\bar{z}^b)' + e^{2U} V_{BH}(z, \bar{z}, p, q), \quad (2.16)$$

supplemented by the constraint

$$(U')^2 + G_{a\bar{b}}(z^a)'(\bar{z}^b)' - e^{2U} V_{BH}(z, \bar{z}, p, q) = 0. \quad (2.17)$$

It should also be mentioned that (2.15) can be expressed as

$$V_{BH}(z, \bar{z}, p, q) = |Z|^2 + |\mathcal{D}_a Z|^2, \quad (2.18)$$

where Z , defined by

$$Z = e^{K/2} (p^I F_I(z) - q_I z^I), \quad (2.19)$$

becomes the central charge at the spatial infinity. Note that \mathcal{D}_a is the Kähler covariant derivative, i.e., $\mathcal{D}_a Z = (\partial_a + \frac{1}{2}\partial_a K)Z$.

3 The exact solutions

Let us consider the simple case of the STU model, whose prepotential is given by

$$F = -\frac{X^1 X^2 X^3}{X^0}. \quad (3.1)$$

Here, we would like to solve the equations of motion for the D0-D4 charge configuration (q_0, p^1, p^2, p^3) . We consider the case in which the moduli fields are complex numbers, i.e.,

$$z^a = x + iy. \quad (a = 1, 2, 3) \quad (3.2)$$

Note, however, that we are considering the special case in which the real and imaginary parts, x and y , are assumed to be common to the three scalar fields. The charges p^a ($a = 1, 2, 3$) are also assumed to be the same, for simplicity:

$$p^1 = p^2 = p^3 = p. \quad (3.3)$$

Under these assumptions, the Kähler potential is computed according to (2.3), and it is found to be

$$e^K = \frac{1}{8y^3}. \quad (3.4)$$

The Kähler metric $G_{a\bar{b}}$ and the quantities ν_{IJ} and μ_{IJ} are given by

$$G_{a\bar{b}} = \frac{1}{4y^2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (3.5)$$

$$\nu_{IJ} = \begin{pmatrix} -2x^3 & x^2 & x^2 & x^2 \\ x^2 & 0 & -x & -x \\ x^2 & -x & 0 & -x \\ x^2 & -x & -x & 0 \end{pmatrix}, \quad (3.6)$$

and

$$\mu_{IJ} = y \begin{pmatrix} -3x^2 - y^2 & x & x & x \\ x & -1 & 0 & 0 \\ x & 0 & -1 & 0 \\ x & 0 & 0 & -1 \end{pmatrix}. \quad (3.7)$$

It is also straightforward to obtain the black hole potential (2.15) and the Lagrangian (2.16) in explicit form:

$$V_{BH} = \frac{1}{2} \left(3p^2 y + \frac{q_0^2}{y^3} + 9p^2 \frac{x^4}{y^3} + 12p^2 \frac{x^2}{y} + 6pq_0 \frac{x^2}{y^3} \right), \quad (3.8)$$

$$\mathcal{L} = (U')^2 + \frac{3}{4y^2} \{ (y')^2 + (x')^2 \} + e^{2U} V_{BH}. \quad (3.9)$$

The equations of motion that we have to solve are obtained by varying \mathcal{L} with respect to U , x and y . This yields

$$U'' = \frac{1}{2} e^{2U} \left(3p^2 y + \frac{q_0^2}{y^3} + 9p^2 \frac{x^4}{y^3} + 12p^2 \frac{x^2}{y} + 6pq_0 \frac{x^2}{y^3} \right), \quad (3.10)$$

$$\left(\frac{x'}{y^2} \right)' = 4e^{2U} \left(3p^2 \frac{x^3}{y^3} + 2p^2 \frac{x}{y} + pq_0 \frac{x}{y^3} \right), \quad (3.11)$$

$$\frac{y''}{y^2} - \frac{(y')^2}{y^3} + \frac{(x')^2}{y^3} = e^{2U} \left(p^2 - \frac{q_0^2}{y^4} - 9p^2 \frac{x^4}{y^4} - 4p^2 \frac{x^2}{y^2} - 6pq_0 \frac{x^2}{y^4} \right). \quad (3.12)$$

Now, in order to rearrange (3.10)–(3.12), we first absorb p and q_0 by introducing M_0 and the new variables ξ and ϕ via

$$M_0^2 = 2\sqrt{p^3 q_0}, \quad x^2 = \frac{q_0}{p} \xi, \quad y = \sqrt{\frac{q_0}{p}} e^\phi. \quad (3.13)$$

Here, we assume $q_0/p > 0$ [11]. We then find that the constraint (2.17) and the equations of motion (3.10), (3.11), and (3.12) are given by

$$(U')^2 + \frac{3}{4}(\phi')^2 + \frac{3(\xi')^2}{16\xi}e^{-2\phi} = \frac{M_0^2}{4}e^{2U} \left\{ 3e^\phi(1 + 4\xi e^{-2\phi}) + e^{-3\phi}(1 + 3\xi)^2 \right\}, \quad (3.14)$$

$$U'' = \frac{M_0^2}{4}e^{2U} \left\{ 3e^\phi(1 + 4\xi e^{-2\phi}) + e^{-3\phi}(1 + 3\xi)^2 \right\}, \quad (3.15)$$

$$\frac{1}{2\xi} \left(\xi' e^{-2\phi} \right)' - \frac{(\xi')^2}{4\xi^2} e^{-2\phi} = M_0^2 e^{2U} \left\{ 4e^{-\phi} + 2e^{-3\phi}(1 + 3\xi) \right\}, \quad (3.16)$$

$$\phi'' + \frac{(\xi')^2}{4\xi} e^{-2\phi} = \frac{M_0^2}{2} e^{2U} \left\{ e^\phi - 4\xi e^{-\phi} - e^{-3\phi}(1 + 3\xi)^2 \right\}. \quad (3.17)$$

Next, we consider the linear combinations of U and ϕ

$$\alpha = U + \frac{1}{2}\phi, \quad \beta = U - \frac{3}{2}\phi, \quad (3.18)$$

with which (3.15) and (3.17) are written

$$\alpha'' + \frac{(\xi')^2}{8\xi} e^{\beta-\alpha} = M_0^2 e^{2\alpha} \left(2\xi e^{\beta-\alpha} + 1 \right), \quad (3.19)$$

$$\beta'' - \frac{3}{4} \left(\xi' e^{\beta-\alpha} \right)' = M_0^2 e^{2\beta} (3\xi + 1), \quad (3.20)$$

with the help of (3.16). Also, to eliminate ξ' , we combine (3.14) and (3.19):

$$3(\alpha')^2 + (\beta')^2 - 6\alpha'' = M_0^2 \left\{ -3e^{2\alpha} + e^{2\beta}(1 + 3\xi)^2 \right\}. \quad (3.21)$$

We now postulate that the solution $\alpha(\tau)$ coincides with what we would have obtained in the case $\xi = 0$. In other words, we assume

$$e^{-\alpha} = \alpha_0 - M_0\tau, \quad (3.22)$$

where α_0 is an arbitrary constant. This postulate enables us to write (3.19) and (3.21) as the first-order differential equations

$$\xi' = 4M_0 e^\alpha \xi = \frac{4M_0 \xi}{\alpha_0 - M_0\tau}, \quad \beta' = M_0 e^\beta (3\xi + 1). \quad (3.23)$$

In fact, we can confirm that ξ and β given by (3.23) also satisfy the second-order equation (3.20). It is now almost trivial to work out the solution

$$\xi = \frac{\gamma_0^2}{(\alpha_0 - M_0\tau)^4}, \quad (3.24)$$

$$e^{-\beta} = (\beta_0 - M_0\tau) - \frac{\gamma_0^2}{(\alpha_0 - M_0\tau)^3}. \quad (3.25)$$

Here, β_0 and γ_0 are arbitrary constants. Combining (3.22) and (3.25), we arrive at

$$e^{2\phi} = e^{\alpha-\beta} = \frac{\beta_0 - M_0\tau}{\alpha_0 - M_0\tau} - \frac{\gamma_0^2}{(\alpha_0 - M_0\tau)^4}, \quad (3.26)$$

$$e^{-4U} = e^{-3\alpha-\beta} = (\alpha_0 - M_0\tau)^3(\beta_0 - M_0\tau) - \gamma_0^2. \quad (3.27)$$

We can easily confirm that (3.24), (3.26), and (3.27) are consistent with the original equations (3.14)–(3.17).

At the spatial infinity ($\tau \rightarrow 0$), we set $U(0) = 0$ and $U'(0) = M$, where M is the black hole mass. These yield the constraint

$$\alpha_0^3\beta_0 - \gamma_0^2 = 1, \quad (3.28)$$

together with the black hole mass

$$M = \frac{M_0}{4} (\alpha_0^3 + 3\alpha_0^2\beta_0). \quad (3.29)$$

This deviates from M_0 in a manner that depends on the initial conditions for the moduli fields at the spatial infinity. It should also be remarked that our solution is non-BPS, because (2.19) in our case is

$$Z = \frac{1}{\sqrt{8y^3}} \{-3p(x+iy)^2 - q_0\}, \quad (3.30)$$

and at the spatial infinity we have

$$\lim_{\tau \rightarrow 0} |Z| = \frac{M_0}{4} \sqrt{(\alpha_0^3 + 3\alpha_0^2\beta_0)^2 - 12\alpha_0^2} \neq M. \quad (3.31)$$

It is illuminating to express our solution in terms of the harmonic functions

$$H = \frac{p}{M_0} (\alpha_0 - M_0\tau), \quad \tilde{H}_0 = \frac{q_0}{M_0} (\beta_0 - M_0\tau). \quad (3.32)$$

In terms of these, (3.24), (3.26), and (3.27) can be rewritten as

$$z = \pm \sqrt{\frac{q_0}{p}} \sqrt{\xi} + i \sqrt{\frac{q_0}{p}} e^\phi = \pm \frac{\gamma_0}{2H^2} + i \sqrt{\frac{\tilde{H}_0}{H} - \frac{\gamma_0^2}{4H^4}}, \quad (3.33)$$

$$e^{-2U} = \sqrt{4H^3\tilde{H}_0 - \gamma_0^2}. \quad (3.34)$$

Note that the moduli fields are attracted to the purely imaginary number

$$z|_{\text{horizon}} = i \sqrt{\frac{q_0}{p}} \quad (3.35)$$

at the horizon ($\tau \rightarrow -\infty$). Apparently, the attractor mechanism is effective, as (3.35) is independent of the initial conditions, α_0 , β_0 , and γ_0 . We also note that the black hole potential V_{BH} is equal to M_0^2 at the horizon, and the black hole entropy S_{BH} is given in terms of the charges carried by the black hole alone: $S_{BH} = \pi V_{BH}|_{\text{horizon}} = \pi M_0^2 = 2\pi\sqrt{p^3 q_0}$.

Finally, the following fact should be pointed out. If we replace the charges in (3.35) by the corresponding harmonic functions (3.32), then we obtain $i\sqrt{\tilde{H}_0/H}$. The solution (3.33), however, possesses the additional parameter γ_0 , which comes from the initial condition at the infinity for the real part of the moduli fields.

4 Summary

In the present paper we have discussed four-dimensional $\mathcal{N} = 2$ supergravity from the Type IIA superstring, which is described by (2.1). We have solved the equations of motion for the STU model (3.1) with D0-D4 brane charges, and our solutions are summarized in (3.33) and (3.34). It should be stressed that our moduli fields are complex, in contrast to those of previous works. This shows that even if we replace the charges in (3.35) by the corresponding harmonic functions, we would not get back the general solutions (3.33) for $\gamma_0 \neq 0$.

As we see in (3.35), the moduli fields are attracted at the horizon to a value determined by only the charges of the black hole. In other words, they are independent of the values of α_0 , β_0 , and γ_0 . Note that our solutions are for the non-BPS black hole, as mentioned in (3.31).

Throughout the present paper, we have been mostly concerned with the analytic behavior of the moduli fields in the whole τ -space. It is known that for BPS black holes we can avoid the problem of solving the differential equations themselves to determine the τ -space behavior. In that case, it is necessary only to solve the algebraic “generalized stabilization equation,” in which all of the non-vanishing charges (q_0, q_a, p^0, p^a) in the attractor equation are replaced by the corresponding harmonic functions (see Appendix A). For non-BPS black holes, however, it is yet to be confirmed that such a simple algebraic prescription is equivalent to solving the second-order differential equations. We hope our solutions for the D0-D4 brane system with complex moduli fields is useful for gaining insight into this and related problems.

After completing the present work, the authors were informed of the recent interesting paper by Cardoso et al [21]. They made use of the first-order flow equation [17] and have obtained exact solutions for complex scalar fields in the whole space.

Acknowledgements

We would like to thank Dr. Yoshifumi Hyakutake for useful discussions. We are also grateful to Professor Gabriel Cardoso for calling our attention to [21]. The work of T.K. is partially supported by a Grant-in-Aid from the Ministry of Education (No. 19034006). K.H. is supported in part by JSPS Research Fellowship for Young Scientists.

A The Generalized Stabilization Equation and the BPS full solution

We would like to supplement the analysis of extremal BPS black holes given in the main text with a treatment of the BPS attractor equations. The purpose of this calculation is to show that if we replace the charges in the double extremal solutions by the corresponding harmonic functions, then we obtain solutions to the first-order BPS condition. As a byproduct of this analysis, we are able to incorporate the real part of the moduli fields.

The BPS black hole solution with non-vanishing charges (p^a, p^0, q_a, q_0) ($a = 1, 2, \dots, N_V$) is derived from the generalized stabilization equation

$$i \begin{pmatrix} H^0 \\ H^a \\ \tilde{H}_0 \\ \tilde{H}_a \end{pmatrix} = e^{K/2} \left\{ \bar{\Sigma} \begin{pmatrix} 1 \\ z^a \\ F_0 \\ F_a \end{pmatrix} - \Sigma \begin{pmatrix} 1 \\ \bar{z}^a \\ \bar{F}_0 \\ \bar{F}_a \end{pmatrix} \right\}, \quad (\text{A.1})$$

which has been obtained by replacing the charges in the BPS attractor equation by the corresponding harmonic functions [5], [6], [19]. Here, the harmonic functions are defined by

$$H^a = h^a - p^a \tau, \quad H^0 = h^0 - p^0 \tau, \quad \tilde{H}_a = \tilde{h}_a - q_a \tau, \quad \tilde{H}_0 = \tilde{h}_0 - q_0 \tau. \quad (\text{A.2})$$

We have also substituted the harmonic functions for the charges in (2.19), and thereby defined the following:

$$\Sigma \equiv e^{K/2} (H^I F_I - \tilde{H}_I z^I). \quad (\text{A.3})$$

The solution of the metric is given by (2.8) with

$$e^{-2U} = |\Sigma|^2. \quad (\text{A.4})$$

In addition, we have to impose the condition of the asymptotic flatness of the metric at the spatial infinity,

$$e^{-2U(\tau=0)} = 1, \quad (\text{A.5})$$

and the constraint

$$h^I q_I = \tilde{h}_I p^I. \quad (\text{A.6})$$

We now confine ourselves to the $N_V = 3$ case and adopt the prepotential (3.1). We further assume that p^a , q_a , h^a , and \tilde{h}_a are common to the three scalar fields, i.e.,

$$H^1 = H^2 = H^3 = H \equiv h - p\tau, \quad (\text{A.7})$$

$$\tilde{H}_1 = \tilde{H}_2 = \tilde{H}_3 = \tilde{H} \equiv \tilde{h} - q\tau. \quad (\text{A.8})$$

We also assume that the three scalar functions take the same forms as in (3.2). It is then straightforward to solve the algebraic equation (A.1), and we get

$$x = -\frac{1}{2} \frac{H\tilde{H} + H^0\tilde{H}_0}{H^2 + \tilde{H}H^0}, \quad (\text{A.9})$$

$$y = \sqrt{\frac{\tilde{H}^2 - H\tilde{H}_0}{H^2 + \tilde{H}H^0} - \frac{1}{4} \left(\frac{H\tilde{H} + H^0\tilde{H}_0}{H^2 + \tilde{H}H^0} \right)^2}. \quad (\text{A.10})$$

According to (A.4), the metric is given in terms of

$$e^{-2U} = 2y(H^2 + \tilde{H}H^0). \quad (\text{A.11})$$

We have confirmed by explicit calculation that the solutions (A.9), (A.10), and (A.11) in fact satisfy the first-order BPS condition [20],

$$U' = e^U |Z|, \quad (z^a)' = e^U G^{a\bar{b}} \mathcal{D}_{\bar{b}} \bar{Z} \frac{Z}{|Z|}. \quad (\text{A.12})$$

A remark is in order with regard to the D0-D4 system, i.e., $p^0 = 0$, $q_a = 0$ and $q_0/p < 0$. In this case, (A.9) and (A.10) are

$$x = -\frac{1}{2} \frac{H\tilde{h} + h^0\tilde{H}_0}{H^2 + \tilde{h}h^0}, \quad y = \sqrt{\frac{\tilde{h}^2 - H\tilde{H}_0}{H^2 + \tilde{h}h^0} - \frac{1}{4} \left(\frac{H\tilde{h} + h^0\tilde{H}_0}{H^2 + \tilde{h}h^0} \right)^2}, \quad (\text{A.13})$$

and at the horizon, these become

$$\lim_{\tau \rightarrow -\infty} x = 0, \quad \lim_{\tau \rightarrow -\infty} y = \sqrt{-\frac{q_0}{p}}. \quad (\text{A.14})$$

It is worth mentioning that the solutions (A.13) have more parameters than what we would obtain by replacing the charges in (A.14) by the corresponding harmonic functions. This is similar to the situation for non-BPS black holes discussed in §3.

B The Non-Extremal Black Holes

A close inspection of the black hole potential V_{BH} reveals that the attractor mechanism is ineffective in some cases, in particular for non-extremal black holes [7], [11], [12]. This fact is confirmed in [7], in which explicit solutions in a particular model are employed. For the sake of completeness, here we present similar analysis adapted to the general prepotential

$$F(X) = D_{abc} \frac{X^a X^b X^c}{X^0}. \quad (a, b, c = 1, \dots, N_V) \quad (\text{B.1})$$

The fact that the attractor mechanism is ineffective is shown here for non-extremal black holes with D0-D4 brane charges on the basis of the exact solutions.

We begin with the metric ansatz for the non-extremal case,

$$ds^2 = -e^{2U(\tau)}(dt)^2 + e^{-2U(\tau)} \left\{ \frac{c^4}{\sinh^4 c\tau} (d\tau)^2 + \frac{c^2}{\sinh^2 c\tau} d\Omega^2 \right\}, \quad (\text{B.2})$$

where a non-vanishing value of c implies that this is the non-extremal black hole. We use the general prepotential (B.1) and consider the D0-D4 brane charges (q_0, p^a) , while $q_a = p^0 = 0$. We also confine ourselves to purely imaginary moduli fields and set

$$z^a = ip^a \sqrt{\frac{q_0}{D}} e^\phi, \quad (\text{for } q_0 < 0) \quad (\text{B.3})$$

$$z^a = ip^a \sqrt{\frac{-q_0}{D}} e^\phi, \quad (\text{for } q_0 > 0) \quad (\text{B.4})$$

where $D = D_{abc} p^a p^b p^c < 0$. Note that in the extremal limit ($c \rightarrow 0$), (B.3) corresponds to the BPS case and (B.4) to the non-BPS case [11].

The equations of motion are

$$(U')^2 + \frac{3}{4}(\phi')^2 = \frac{M_0^2}{4}e^{2U} \left(3e^\phi + e^{-3\phi} \right) + c^2, \quad (\text{B.5})$$

$$U'' = \frac{M_0^2}{4}e^{2U} \left(3e^\phi + e^{-3\phi} \right), \quad (\text{B.6})$$

$$\phi'' = \frac{M_0^2}{2}e^{2U} \left(e^\phi - e^{-3\phi} \right) \quad (\text{B.7})$$

for both (B.3) and (B.4). Here, we have defined

$$M_0^2 = 2\sqrt{q_0 D}, \quad (\text{for } q_0 < 0) \quad (\text{B.8})$$

$$M_0^2 = 2\sqrt{-q_0 D}. \quad (\text{for } q_0 > 0) \quad (\text{B.9})$$

As we see, the effects of the non-extremal nature of the black hole appear only in (B.5).

We next define new variables α and β as in (3.18). Their equations of motion are easily found to be

$$\alpha'' = M_0^2 e^{2\alpha}, \quad \beta'' = M_0^2 e^{2\beta}. \quad (\text{B.10})$$

The constraint equation is also expressed as

$$\frac{3}{4}(\alpha')^2 + \frac{1}{4}(\beta')^2 = \frac{M_0^2}{4} \left(3e^{2\alpha} + e^{2\beta} \right) + c^2. \quad (\text{B.11})$$

The equations in (B.10) are of the Toda-type, and their solutions are

$$e^{-\alpha} = \frac{\sinh[A(\alpha_0 - M_0\tau)]}{A}, \quad e^{-\beta} = \frac{\sinh[B(\beta_0 - M_0\tau)]}{B}. \quad (\text{B.12})$$

Here, A , B , α_0 , and β_0 are constants of integration. These solutions, in turn, give U and ϕ through the following:

$$e^{-U} = \left(\frac{\sinh[A(\alpha_0 - M_0\tau)]}{A} \right)^{3/4} \left(\frac{\sinh[B(\beta_0 - M_0\tau)]}{B} \right)^{1/4}, \quad (\text{B.13})$$

$$e^\phi = \left(\frac{A \sinh[B(\beta_0 - M_0\tau)]}{B \sinh[A(\alpha_0 - M_0\tau)]} \right)^{1/2}. \quad (\text{B.14})$$

The constraint equation (B.11) imposes a relation between the integration constants A and B :

$$\frac{M_0^2}{4} (3A^2 + B^2) = c^2. \quad (\text{B.15})$$

We thus see that in the extremal case ($c \rightarrow 0$), which implies the limits $A \rightarrow 0$ and $B \rightarrow 0$, the solutions reduce to

$$e^{-U} \rightarrow (\alpha_0 - M_0\tau)^{3/4}(\beta_0 - M_0\tau)^{1/4}, \quad (\text{B.16})$$

$$e^\phi \rightarrow \sqrt{\frac{\beta_0 - M_0\tau}{\alpha_0 - M_0\tau}}. \quad (\text{B.17})$$

The mass of the black hole M is defined by the asymptotic form of the metric as $\tau \rightarrow 0$:

$$e^{-U(\tau)} \rightarrow 1 - M\tau. \quad (\text{B.18})$$

This provides the constraint

$$\left(\frac{\sinh A\alpha_0}{A}\right)^3 \left(\frac{\sinh B\beta_0}{B}\right) = 1, \quad (\text{B.19})$$

together with the formula for the mass

$$M = \frac{M_0}{4} \left\{ \left(\frac{\sinh A\alpha_0}{A}\right)^3 \cosh B\beta_0 + 3 \left(\frac{\sinh A\alpha_0}{A}\right)^2 \frac{\sinh B\beta_0}{B} \cosh A\alpha_0 \right\}. \quad (\text{B.20})$$

The minimum value of M is

$$\frac{M_0}{4} (\alpha_0^3 + 3\alpha_0^2\beta_0), \quad (\text{B.21})$$

and this corresponds to the extremal case, i.e., $A = B = 0$.

The central charge can also be computed as

$$Z = \frac{1}{4} M_0 \left(3e^{\phi/2} \pm e^{-3\phi/2} \right), \quad (\text{B.22})$$

where $+$ and $-$ correspond to $q_0 < 0$ and $q_0 > 0$, respectively. We can easily confirm that $M \neq |Z|_{\tau=0}$; i.e., our non-extremal solution is non-BPS.

The behavior of the moduli fields at the horizon ($\tau \rightarrow -\infty$) is seen from (B.14):

$$e^\phi \rightarrow \sqrt{\frac{A}{B}} e^{(B\beta_0 - A\alpha_0)/2} e^{(A-B)M_0\tau/2}. \quad (\text{B.23})$$

This is divergent if $A < B$ and vanishing if $A > B$. If we impose the condition that the moduli fields are regular and non-vanishing, then we have to additionally impose the condition $A = B$. Apparently, even for $A = B$, the value of the moduli fields at the horizon depends on the arbitrary constants α_0 and β_0 . Thus, the attractor mechanism is ineffectual.

Finally, let us evaluate the entropy of the black hole. For $A = B$, the limiting behavior

$$e^{-2U} \frac{c^2}{\sinh^2 c\tau} \rightarrow \left(\frac{e^{A(\alpha_0 - M_0\tau)}}{2A} \right)^{3/2} \left(\frac{e^{A(\beta_0 - M_0\tau)}}{2A} \right)^{1/2} \frac{4c^2}{e^{-2c\tau}} \quad (\text{B.24})$$

$$= M_0^2 e^{A(3\alpha_0 + \beta_0)/2} \quad (\text{B.25})$$

can be derived for $\tau \rightarrow -\infty$. Here, use has been made of the relation $M_0^2 A^2 = c^2$, which comes from (B.15) by setting $A = B$. Thus, the Bekenstein-Hawking entropy,

$$\frac{1}{4} \times (\text{Area}) = \pi M_0^2 e^{A(3\alpha_0 + \beta_0)/2}, \quad (\text{B.26})$$

depends on the constants α_0 and β_0 if $A \neq 0$.

References

- [1] S. Ferrara, R. Kallosh, A. Strominger, Phys. Rev. D **52** (1995), 5412; hep-th/9508072.
- [2] S. Ferrara and R. Kallosh, Phys. Rev. D **54** (1996), 1514; hep-th/9602136; Phys. Rev. D **54** (1996), 1525; hep-th/9603090; G. W. Gibbons, R. Kallosh and B. Kol, Phys. Rev. Lett. **77** (1996), 4992; hep-th/9607108; S. Ferrara, G. W. Gibbons and R. Kallosh, Nucl. Phys. B **500** (1997), 75; hep-th/9702103.
- [3] A. Strominger, Phys. Lett. B **383** (1996), 39; hep-th/9602111.
- [4] M. Cvetič and A. A. Tseytlin, Phys. Rev. D **53** (1996), 5619; [Errata; **55** (1997), 3907]; hep-th/9512031; M. Cvetič and C. M. Hull, Nucl. Phys. B **480** (1996), 296; hep-th/9606193.
- [5] W. A. Sabra, Mod. Phys. Lett. A **12** (1997), 2585; hep-th/9703101; Nucl. Phys. B **510** (1998), 247; hep-th/9704147; K. Behrndt, D. Lüst and W. A. Sabra, Nucl. Phys. B **510** (1998) 264; hep-th/9705169.
- [6] F. Denef, J. High Energy Phys. **08** (2000), 050; hep-th/0005049; F. Denef, B. Greene and M. Raugas, J. High Energy Phys. **05** (2001), 012; hep-th/0101135; B. Bates and F. Denef, hep-th/0304094.
- [7] K. Goldstein, N. Iizuka, R. P. Jena and S. P. Trivedi, Phys. Rev. D **72** (2005), 124021; hep-th/0507096.

- [8] D. Astefanesei, K. Goldstein and S. Mahapatra, hep-th/0611140.
- [9] A. Sen, J. High Energy Phys. **09** (2005), 038; hep-th/0506177; J. High Energy Phys. **03** (2006), 008; hep-th/0508042; D. Astefanesei, K. Goldstein, R. P. Jena, A. Sen and P. Trivedi, J. High Energy Phys. **10** (2006) 058; hep-th/0606244.
- [10] M. R. Garousi and A. Ghodsi, J. High Energy Phys. **05** (2007) 043; hep-th/0703260; arXiv:0705.2149.
- [11] P. K. Tripathy and S. P. Trivedi, J. High Energy Phys. **03** (2006), 022; hep-th/0511117.
- [12] R. Kallosh, J. High Energy Phys. **12** (2005), 022; hep-th/0510024.
R. Kallosh, N. Sivanandam and M. Soroush, J. High Energy Phys. **03** (2006), 060; hep-th/0602005.
- [13] A. Misra and P. Shukla, arXiv:0707.0105; P. Kaura and A. Misra, Fortschr. Phys. **54** (2006), 1109; hep-th/0607132.
- [14] R. Kallosh, N. Sivannandam and M. Soroush, Phys. Rev. D **74** (2006), 065008; hep-th/0606263.
- [15] R. Kallosh, hep-th/0603003.
- [16] A. Dabholkar, A. Sen, and S. P. Trivedi, J. High Energy Phys. **01** (2007), 096; hep-th/0611143.
- [17] A. Ceresole and G. Dall'Agata, J. High Energy Phys. **03** (2007), 110; hep-th/0702088.
- [18] K. Saraikin and C. Vafa, hep-th/0703214.
- [19] G. L. Cardoso, B. de Wit, J. Kappeli and T. Mohaupt, J. High Energy Phys. **12** (2000), 019; hep-th/0009234; T. Mohaupt, Fortschr. Phys. **49** (2001), 3; hep-th/0007195.
- [20] S. Bellucci, S. Ferrara and A. Marrani, *Supersymmetric Mechanics* vol. 2 (Springer, 2006)
- [21] G. L. Cardoso, A. Ceresole, G. Dall'Agata, J. M. Oberreuter and J. Perz, J. High Energy Phys. **10** (2007), 110; arXiv:0706.3373.